

More than you ever wanted to know about

The Prisoner's Dilemma

Tony Boyles

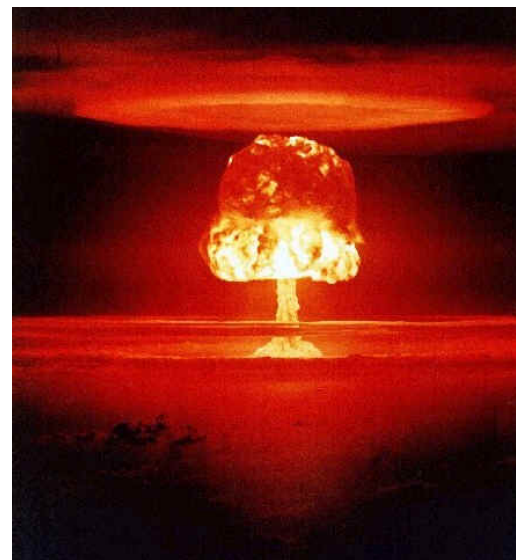
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What we're doing here

The Three Models of the Prisoner's Dilemma

- Normal Form Game
- Game of Mixed Strategy
- Repeated Game

...As Applied to Nuclear Strategy,
And what's really goes on.



The Prisoner's Dilemma



The Prisoner's Dilemma

Player 2

Player 1

	Cooperate	Defect
Cooperate	3, 3	1, 4
Defect	4, 1	2, 2

The Prisoner's Dilemma

Player 2

	Cooperate	Defect
Cooperate	3, 3	1, 4*
Defect	*4, 1	*2, 2*

Player 1



Nash Equilibrium

The Prisoner's Dilemma, Generalized

Player 2

Player 1

	Cooperate	Defect
Cooperate	Good	Worst, Best
Defect	Best, Worst	Bad

The Prisoner's Dilemma, Generalized

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	C, C	A, D
	Defect	D, A	B, B

If we know that (Defect, Defect) is the Nash Equilibrium, for what values of A, B, C, and D is this a Prisoner's Dilemma?

The Prisoner's Dilemma, Generalized

Player 2

Player 1

	Cooperate	Defect
Cooperate	C, C	A, D
Defect	D, A	B, B

$D > C$

$C > B$

$B > A$

So, $D > C > B > A$

Is This A Prisoner's Dilemma?

Player 2

Player 1

	Cooperate	Defect
Cooperate	4, 4	2, 13
Defect	13, 2	1, 1

Is This A Prisoner's Dilemma?

Player 2

		Cooperate	Defect
Player 1	Cooperate	4, 4	*2, 13*
	Defect	*13, 2*	1, 1

Nope!

Is Nuclear War a Prisoner's Dilemma?



Is Nuclear War a Prisoner's Dilemma?

	Cooperate	Defect
Cooperate	0, 0	-100, 10
Defect	10, -100	-90, -90

Is Nuclear War a Prisoner's Dilemma?

	Cooperate	Defect
Cooperate	0, 0	-100, 10
Defect	10, -100	-90, -90

Substantive Assumptions:

- Threat is a nontrivial factor
- Decision makers view themselves as better off if everyone suffers

Could a Nuclear War have Happened?

		p	(1-p)
		Cooperate	Defect
q	Cooperate	0, 0	-100, 10
(1-q)	Defect	10, -100	-90, -90

Could a Nuclear War have Happened?

		p	(1-p)
		Cooperate	Defect
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$$E(\text{Cooperate}) = 0(p) + -100(1-p)$$

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$$-100 + 100p = 10p - 90 + 90p$$

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$$-10 = 0$$

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$$-100 + 100p = 10p - 90 + 90p$$

$$-10 = 0$$



There does not exist any possible probability value which satisfies this relationship!

What if we could assign a probability?



What if we could assign a probability?

$$\Pr(\text{No Attack today}) = p = .999$$



What if we could assign a probability?

$\Pr(\text{No Attack today}) = p = .999$

$\Pr(\text{No Attack today AND tomorrow}) = .999^2$



What if we could assign a probability?

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What if we could assign a probability?

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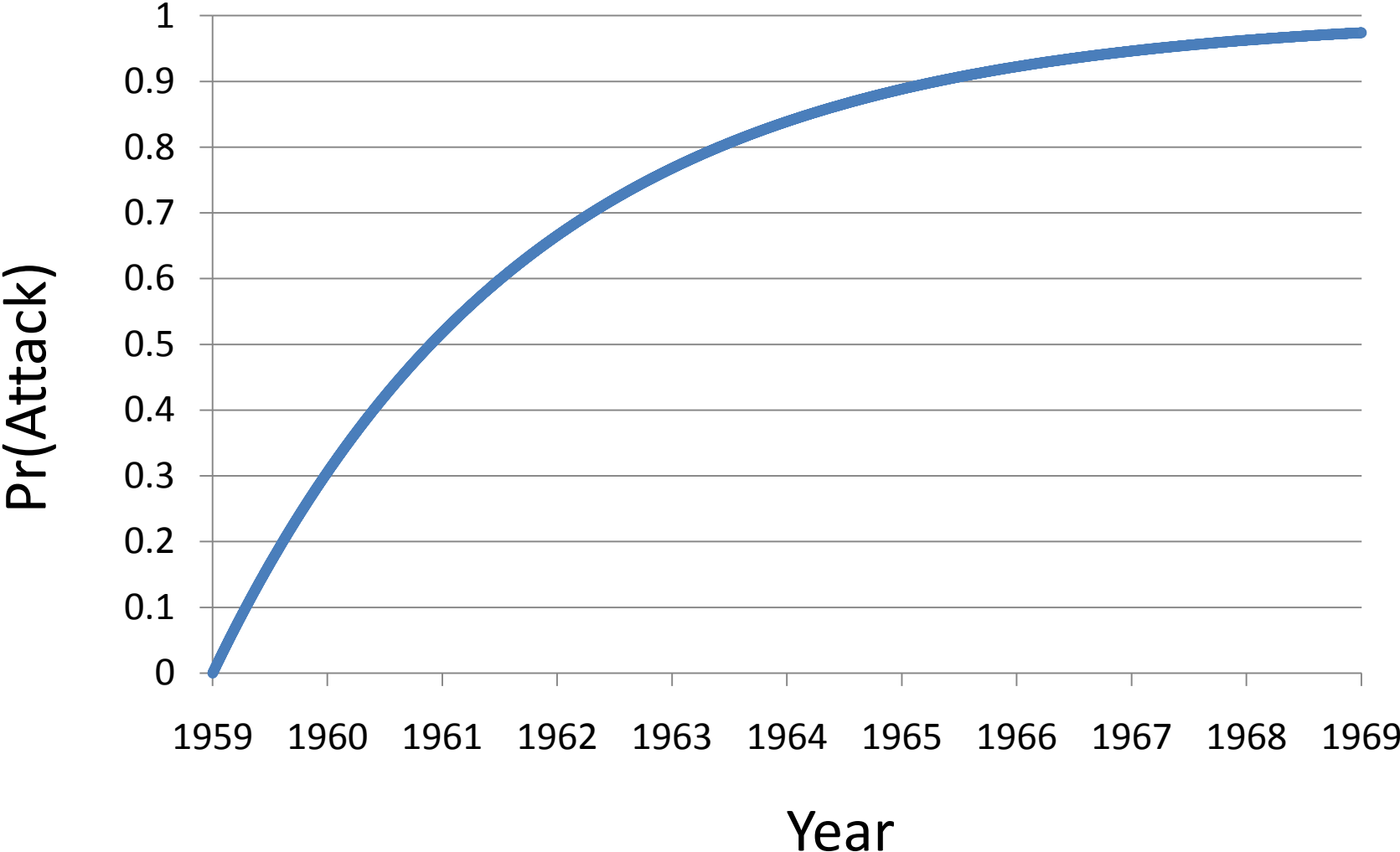
$$\Pr(\text{No Attack today AND tomorrow}) = .999^2$$

$$\Pr(\text{No Attack over } n \text{ days}) = (.999)^n$$

$$\Pr(\text{Attack over } n \text{ days}) = 1 - (.999)^n$$



Probability of a Nuclear Attack



If Probability Theory doesn't help...

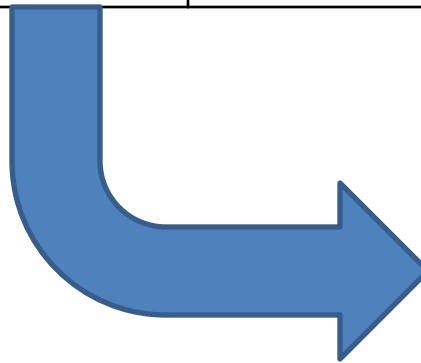
What if it's a repeated game?

	Cooperate	Defect
Cooperate	0, 0	-100, 10
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If Probability Theory doesn't help...

What if it's a repeated game?

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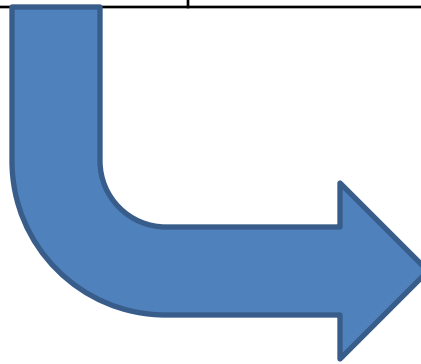
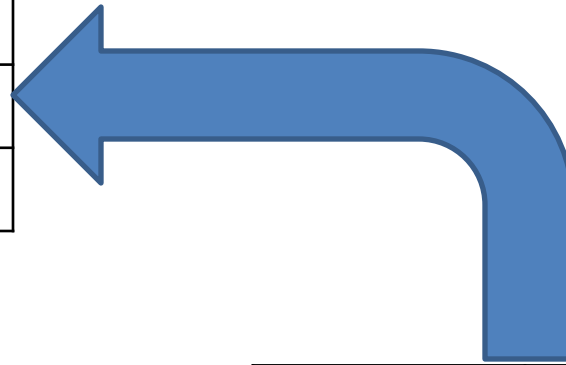


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If Probability Theory doesn't help...

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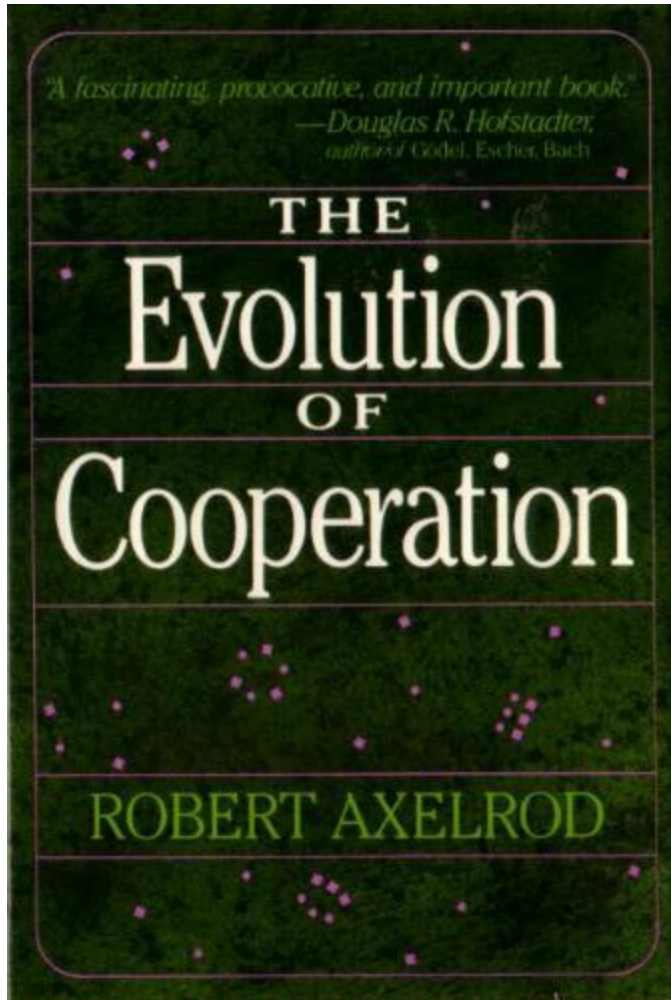
If Probability Theory doesn't help...

What if it's a repeated game?

Grim Trigger?



If Probability Theory doesn't help...

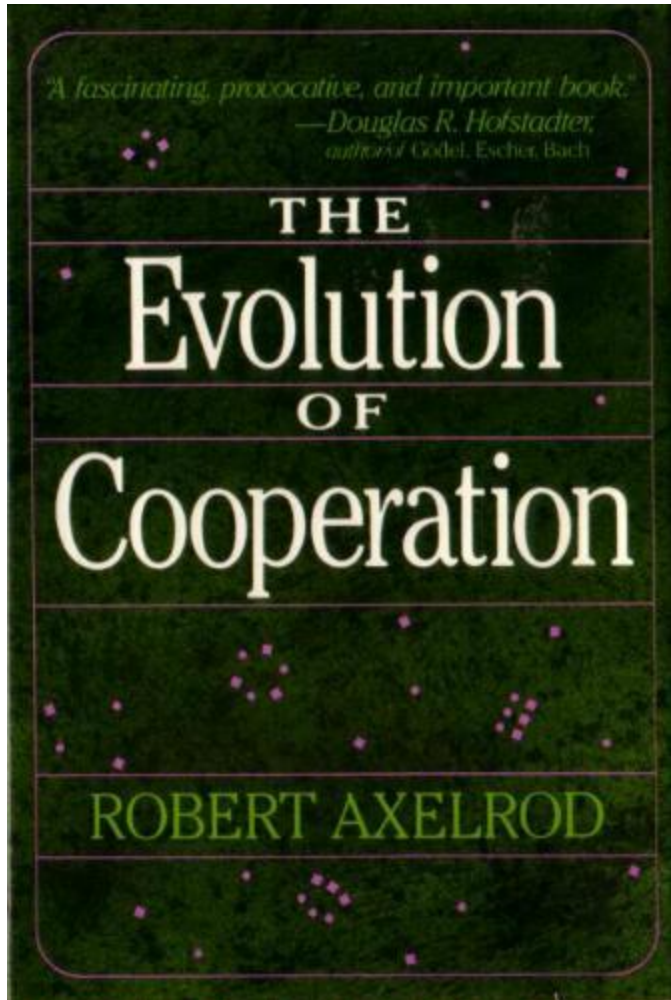


What if it's a repeated game?

Grim Trigger?

Tit-for-tat!

If Probability Theory doesn't help...



What if it's a repeated game?

Grim Trigger?

Tit-for-tat!

Tit-for-two-tats?

The Prisoner's Dilemma, Generalized

Player 2

	Cooperate	Defect
Cooperate	Good	Worst, Best
Defect	Best, Worst	Bad

Player 1

The Prisoner's Dilemma, Generalized

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	Good	Worst, Best
	Defect	Best, Worst	Bad

Substantive Assumptions:

- Threat is a nontrivial factor
- Decision makers view themselves as better off if everyone suffers

The Nuclear Prisoner's Dilemma?

Player 2

	Cooperate	Defect
Cooperate		
Defect		

Player 1

The Nuclear Prisoner's Dilemma?

Player 2

Player 1

	Cooperate	Defect
Cooperate	0, 0	-100, 0
Defect	0, -100	-100, -100

The Nuclear Game

Player 2

Player 1

	Cooperate	Defect
Cooperate	*0, 0*	*-100, 0*
Defect	*0, -100*	*-100, -100*

The Nuclear Game

Player 2

Player 1

	Cooperate	Defect
Cooperate	*0, 0*	
Defect		*-100, -100*

And Deterrence is Born!



Thank you very much.

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